Transformations and Similarity

? ESSENTIAL QUESTION

How can you use dilations and similarity to solve real-world problems?
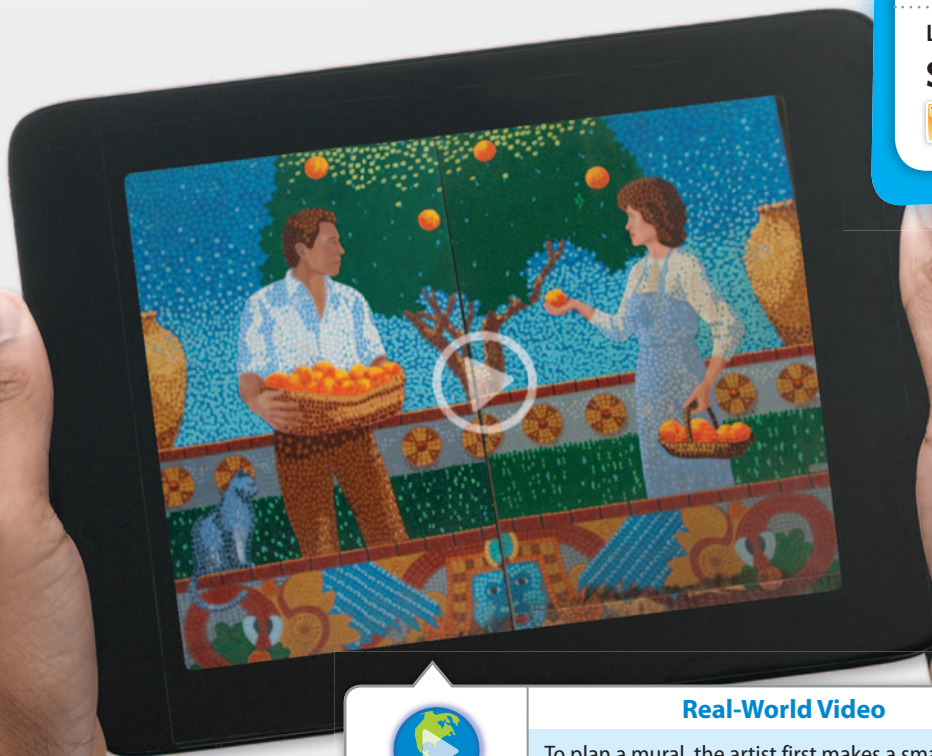
Real-World Video

To plan a mural, the artist first makes a smaller drawing showing what the mural will look like. Then the image is enlarged by a scale factor on the mural canvas. This enlargement is called a dilation.

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.

Simplify Ratios

EXAMPLE
$\dfrac{35}{21} = \dfrac{35 \div 7}{21 \div 7}$

$= \dfrac{5}{3}$

To write a ratio in simplest form, find the greatest common factor of the numerator and denominator. Divide the numerator and denominator by the GCF.

Write each ratio in simplest form.

1. $\dfrac{6}{15}$ _____

2. $\dfrac{8}{20}$ _____

3. $\dfrac{30}{18}$ _____

4. $\dfrac{36}{30}$ _____

Multiply with Fractions and Decimals

EXAMPLE
$2\frac{3}{5} \times 20$

$= \dfrac{13 \times 20}{5 \times 1}$

$= \dfrac{13 \times \overset{4}{\cancel{20}}}{\underset{1}{\cancel{5}} \times 1}$

$= 52$

Write numbers as fractions and multiply.

Simplify.

$\begin{array}{r} 68 \\ \times\,4.5 \\ \hline 340 \\ +272 \\ \hline 306.0 \end{array}$
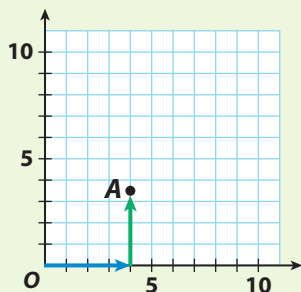
Multiply as you would with whole numbers.

Place the decimal point in the answer based on the total number of decimal places in the two factors.

Multiply.

5. $60 \times \dfrac{25}{100}$

6. 3.5×40

7. 4.4×44

8. $24 \times \dfrac{8}{9}$

_____ _____ _____ _____

Graph Ordered Pairs (First Quadrant)

EXAMPLE

Graph the point A(4, 3.5).
Start at the origin.
Move 4 units right.
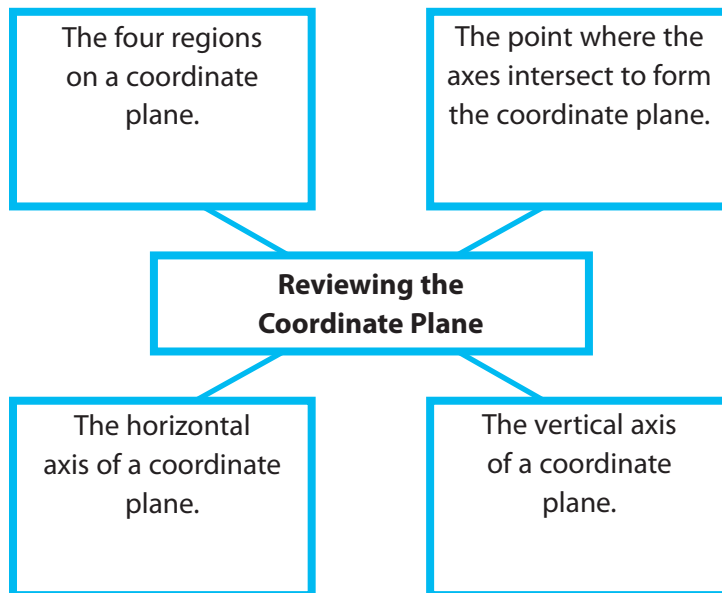Then move 3.5 units up.
Graph point A(4, 3.5).

Graph each point on the coordinate grid above.

9. $B\,(9, 0)$

10. $C\,(2, 7)$

11. $D\,(0, 4.5)$

12. $E\,(6, 2.5)$

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic organizer.
You will put one word in each rectangle.

```
┌─────────────────┐        ┌─────────────────┐
│  The four regions│        │ The point where │
│  on a coordinate │        │ the axes        │
│      plane.      │        │ intersect to    │
│                  │        │ form the        │
│                  │        │ coordinate      │
│                  │        │ plane.          │
└────────┬─────────┘        └────────┬────────┘
         │    ┌──────────────────┐   │
         └────┤  Reviewing the   ├───┘
         ┌────┤ Coordinate Plane ├───┐
         │    └──────────────────┘   │
┌────────┴─────────┐        ┌────────┴────────┐
│  The horizontal  │        │ The vertical    │
│ axis of a        │        │ axis of a       │
│ coordinate       │        │ coordinate      │
│   plane.         │        │   plane.        │
└──────────────────┘        └─────────────────┘
```

Understand Vocabulary

Complete the sentences using the preview words.

1. A figure larger than the original, produced through dilation, is

   an _____.

2. A figure smaller than the original, produced through dilation, is

   a _____.

Active Reading

Key-Term Fold Before beginning the module, create a key-term fold to help you learn the vocabulary in this module. Write the highlighted vocabulary words on one side of the flap. Write the definition for each word on the other side of the flap. Use the key-term fold to quiz yourself on the definitions used in this module.

Vocabulary

Review Words

coordinate plane *(plano cartesiano)*
image *(imagen)*
✔ origin *(origen)*
preimage *(imagen original)*
✔ quadrants *(cuadrante)*
ratio *(razón)*
scale *(escala)*
✔ *x*-axis *(eje x)*
✔ *y*-axis *(eje y)*

Preview Words

center of dilation *(centro de dilatación)*
dilation *(dilatación)*
enlargement *(agranda- miento)*
reduction *(reducción)*
scale factor *(factor de escala)*
similar *(similar)*

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.
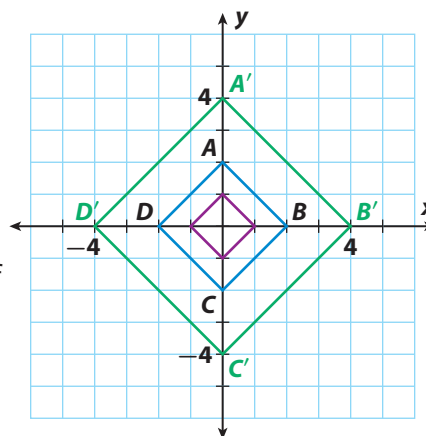

FL 8.G.1.3

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

What It Means to You

You will use an algebraic representation to describe a dilation.

UNPACKING EXAMPLE 8.G.1.3

The blue square *ABCD* is the preimage. Write two algebraic representations, one for the dilation to the green square and one for the dilation to the purple square.

The coordinates of the vertices of the original image are multiplied by 2 for the green square.

Green square: $(x, y) \rightarrow (2x, 2y)$

The coordinates of the vertices of the original image are multiplied by $\frac{1}{2}$ for the purple square.

Purple square: $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y \right)$
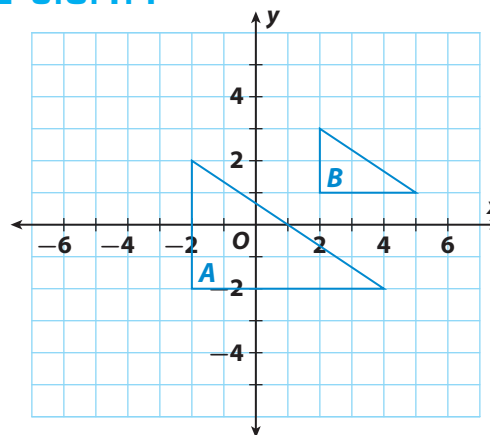

FL 8.G.1.4

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

What It Means to You

You will describe a sequence of transformations between two similar figures.

UNPACKING EXAMPLE 8.G.1.4

Identify a sequence of two transformations that will transform figure *A* into figure *B*.

Dilate with center at the origin by a scale factor of $\frac{1}{2}$.

Then translate right 3 units and up 2 units.

Properties of Dilations

? ESSENTIAL QUESTION

How do you describe the properties of dilations?

## EXPLORE ACTIVITY 1 Real World

FL 8.G.1.4

Exploring Dilations

The missions that placed 12 astronauts on the moon were controlled at the Johnson Space Center in Houston. The toy models at the right are scaled-down replicas of the Saturn V rocket that powered the moon flights. Each replica is a transformation called a **dilation**. Unlike the other transformations you have studied—translations, rotations, and reflections—dilations change the size (but not the shape) of a figure.
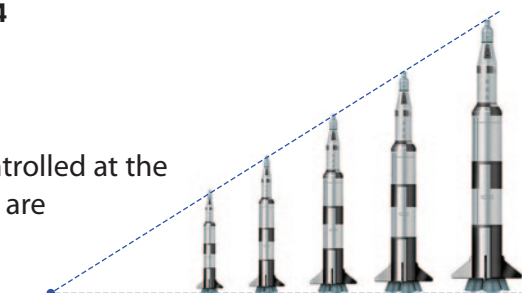
Every dilation has a fixed point called the **center of dilation** located where the lines connecting corresponding parts of figures intersect.

Center of dilation

Triangle *R'S'T'* is a dilation of triangle *RST*. Point *C* is the center of dilation.
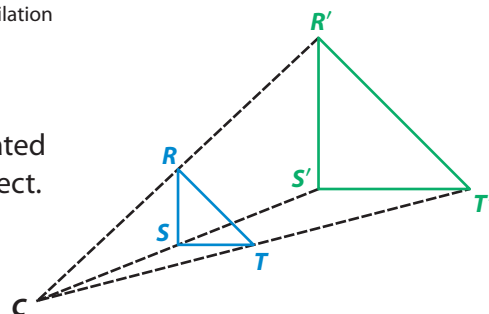
A Use a ruler to measure segments $\overline{CR}$, $\overline{CR'}$, $\overline{CS}$, $\overline{CS'}$, $\overline{CT}$, and $\overline{CT'}$ to the nearest millimeter. Record the measurements and ratios in the table.

CR'	*CR*	$\frac{CR'}{CR}$	*CS'*	*CS*	$\frac{CS'}{CS}$	*CT'*	*CT*	$\frac{CT'}{CT}$

B Write a conjecture based on the ratios in the table.

_____

C Measure and record the corresponding side lengths of the triangles.

R'S'	*RS*	$\frac{R'S'}{RS}$	*S'T'*	*ST*	$\frac{S'T'}{ST}$	*R'T'*	*RT*	$\frac{R'T'}{RT}$

D Write a conjecture based on the ratios in the table.

_____

E Measure the corresponding angles and describe your results.

_____

Reflect

1. Two figures that have the same shape but different sizes are called *similar*. Are triangles *RST* and *R'S'T'* similar? Why or why not?

_____

_____

2. Compare the orientation of a figure with the orientation of its dilation.
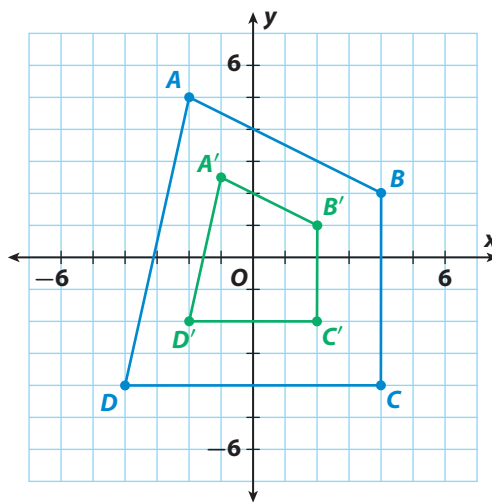
_____

EXPLORE ACTIVITY 2   FL   8.G.1.3

Exploring Dilations on a Coordinate Plane

In this activity you will explore how the coordinates of a figure on a coordinate plane are affected by a dilation.

A Complete the table. Record the *x*- and *y*-coordinates of the points in the two figures and the ratios of the *x*-coordinates and the *y*-coordinates.

Vertex	x	y	Vertex	x	y	Ratio of x-coordinates (A'B'C'D' ÷ ABCD)	Ratio of y-coordinates (A'B'C'D' ÷ ABCD)
A'			A				
B'			B				
C'			C				
D'			D				

B Write a conjecture about the ratios of the coordinates of a dilation image to the coordinates of the original figure.

_____

_____

Reflect

3. In Explore Activity 1, triangle $R'S'T'$ was larger than triangle RST. How is the relationship between quadrilateral $A'B'C'D'$ and quadrilateral $ABCD$ different?

_____

Finding a Scale Factor

As you have seen in the two activities, a dilation can produce a larger figure (an **enlargement**) or a smaller figure (a **reduction**). The **scale factor** describes how much the figure is enlarged or reduced. The scale factor is the ratio of a length of the image to the corresponding length on the original figure.

In Explore Activity 1, the side lengths of triangle $R'S'T'$ were twice the length of those of triangle RST, so the scale factor was 2. In Explore Activity 2, the side lengths of quadrilateral $A'B'C'D'$ were half those of quadrilateral $ABCD$, so the scale factor was 0.5.

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EXAMPLE 1 Real World   FL 8.G.1.4

An art supply store sells several sizes of drawing triangles. All are dilations of a single basic triangle. The basic triangle and one of its dilations are shown on the grid. Find the scale factor of the dilation.

STEP 1 Use the coordinates to find the lengths of the sides of each triangle.

Triangle ABC: $AC = 2$ $CB = 3$

Triangle $A'B'C'$: $A'C' = 4$ $C'B' = 6$
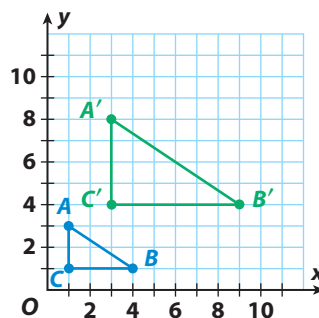
Since the scale factor is the same for all corresponding sides, you can record just two pairs of side lengths. Use one pair as a check on the other.
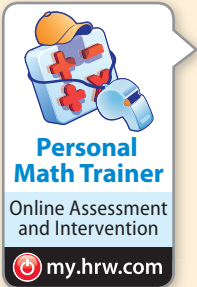
STEP 2 Find the ratios of the corresponding sides.

$\dfrac{A'C'}{AC} = \dfrac{4}{2} = 2$ $\dfrac{C'B'}{CB} = \dfrac{6}{3} = 2$

The scale factor of the dilation is 2.
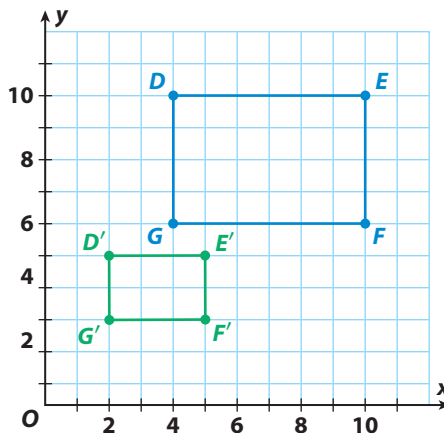
Reflect

4. Is the dilation an enlargement or a reduction? How can you tell?

_____

_____

YOUR TURN

5. Find the scale factor of the dilation.

_____

_____

Math Talk
Mathematical Practices

Which scale factors lead to enlargements? Which scale factors lead to reductions?

Guided Practice

Use triangles *ABC* and *A′B′C′* for 1–5. (Explore Activities 1 and 2, Example 1)

1. For each pair of corresponding vertices, find the ratio of the *x*-coordinates and the ratio of the *y*-coordinates.

ratio of *x*-coordinates = _____

ratio of *y*-coordinates = _____

2. I know that triangle *A′B′C′* is a dilation of triangle *ABC* because the ratios of the corresponding

x-coordinates are _____ and the ratios of the

corresponding *y*-coordinates are _____.

3. The ratio of the lengths of the corresponding sides of triangle *A′B′C′* and

triangle *ABC* equals _____.

4. The corresponding angles of triangle *ABC* and triangle *A′B′C′*

are _____.

5. The scale factor of the dilation is _____.
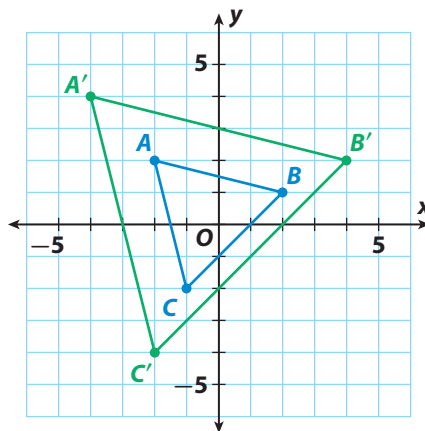
? ESSENTIAL QUESTION CHECK-IN

6. How can you find the scale factor of a dilation?

_____

_____

10.1 Independent Practice

FL 8.G.1.3, 8.G.1.4

For 7–11, tell whether one figure is a dilation of the other or not. Explain your reasoning.

7. Quadrilateral *MNPQ* has side lengths of 15 mm, 24 mm, 21 mm, and 18 mm. Quadrilateral *M'N'P'Q'* has side lengths of 5 mm, 8 mm, 7 mm, and 4 mm.

_____

_____

_____

8. Triangle *RST* has angles measuring 38° and 75°. Triangle *R'S'T'* has angles measuring 67° and 38°. The sides are proportional.

_____

_____

_____

9. Two triangles, Triangle 1 and Triangle 2, are similar.

_____

_____

10. Quadrilateral *MNPQ* is the same shape but a different size than quadrilateral *M'N'P'Q*.

_____

_____

_____

_____

_____

11. On a coordinate plane, triangle *UVW* has coordinates *U*(20, −12), *V*(8, 6), and *W*(−24, −4). Triangle *U'V'W'* has coordinates *U'*(15, −9), *V'*(6, 4.5), and *W'*(−18, −3).

_____

_____

_____

_____

_____

_____

Complete the table by writing "same" or "changed" to compare the image with the original figure in the given transformation.

	Image Compared to Original Figure		
	Orientation	Size	Shape
12. Translation			
13. Reflection			
14. Rotation			
15. Dilation			

16. Describe the image of a dilation with a scale factor of 1.

_____

_____

Identify the scale factor used in each dilation.

17.

_____

18.

_____
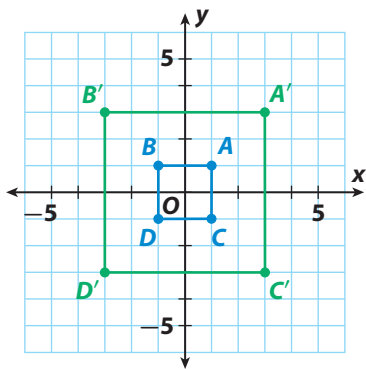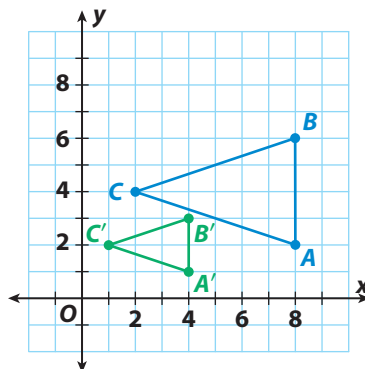
19. Critical Thinking Explain how you can find the center of dilation of a triangle and its dilation.

_____

_____

_____

20. Make a Conjecture

a. A square on the coordinate plane has vertices at $(-2, 2)$, $(2, 2)$, $(2, -2)$, and $(-2, -2)$. A dilation of the square has vertices at $(-4, 4)$, $(4, 4)$, $(4, -4)$, and $(-4, -4)$. Find the scale factor and the perimeter of each square.

_____

_____

b. A square on the coordinate plane has vertices at $(-3, 3)$, $(3, 3)$, $(3, -3)$, and $(-3, -3)$. A dilation of the square has vertices at $(-6, 6)$, $(6, 6)$, $(6, -6)$, and $(-6, -6)$. Find the scale factor and the perimeter of each square.

_____

_____

c. Make a conjecture about the relationship of the scale factor to the perimeter of a square and its image.

_____

_____

Work Area

Algebraic Representations of Dilations

? **ESSENTIAL QUESTION**

How can you describe the effect of a dilation on coordinates using an algebraic representation?

**EXPLORE ACTIVITY 1**   FL   8.G.1.3

Graphing Enlargements

When a dilation in the coordinate plane has the origin as the center of dilation, you can find points on the dilated image by multiplying the x- and y-coordinates of the original figure by the scale factor. For scale factor k, the algebraic representation of the dilation is $(x, y) \rightarrow (kx, ky)$. For enlargements, $k > 1$.

The figure shown on the grid is the preimage. The center of dilation is the origin.
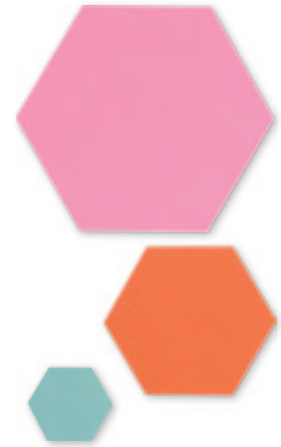
A List the coordinates of the vertices of the preimage in the first column of the table.

Preimage (x, y)	Image (3x, 3y)
(2, 2)	(6, 6)
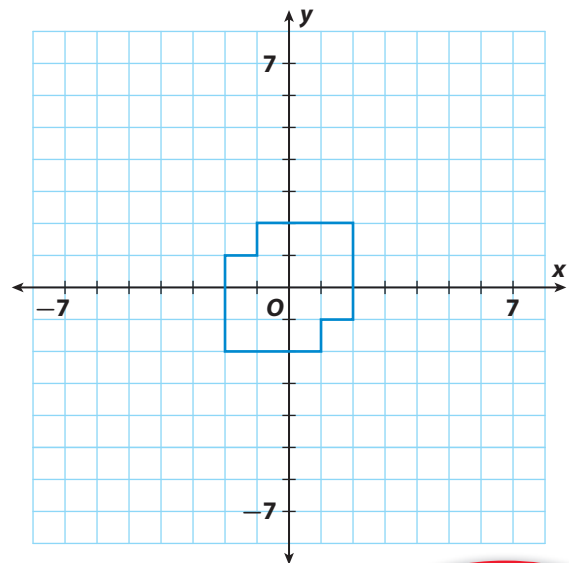
B What is the scale factor for the dilation? _____

C Apply the dilation to the preimage and write the coordinates of the vertices of the image in the second column of the table.

D Sketch the image after the dilation on the coordinate grid.

Math Talk
Mathematical Practices

What effect would the dilation $(x, y) \rightarrow (4x, 4y)$ have on the radius of a circle?

Reflect

1. How does the dilation affect the length of line segments?

_____

_____

2. How does the dilation affect angle measures?

_____

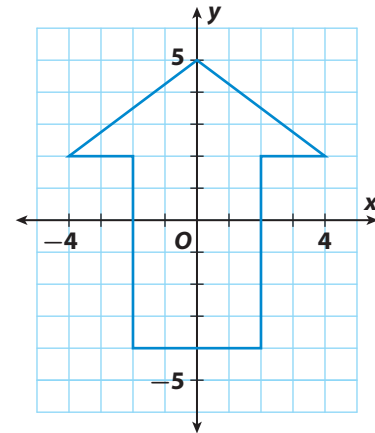**EXPLORE ACTIVITY 2**   FL   8.G.1.3

Graphing Reductions

For scale factors between 0 and 1, the image is smaller than the preimage. This is called a reduction.

The arrow shown is the preimage. The center of dilation is the origin.

A List the coordinates of the vertices of the preimage in the first column of the table.

B What is the scale factor

for the dilation? _____

C Apply the dilation to the preimage and write the coordinates of the vertices of the image in the second column of the table.

Preimage (x, y)	Image $\left(\frac{1}{2}x, \frac{1}{2}y\right)$

D Sketch the image after the dilation on the coordinate grid.

Reflect

3. How does the dilation affect the length of line segments?

_____

4. How would a dilation with scale factor 1 affect the preimage?

_____

Center of Dilation Outside the Image

The center of dilation can be inside *or* outside the original image and the dilated image. The center of dilation can be anywhere on the coordinate plane as long as the lines that connect each pair of corresponding vertices between the original and dilated image intersect at the center of dilation.

EXAMPLE 1

FL   8.G.1.3

Graph the image of △ABC after a dilation with the origin as its center and a scale factor of 3. What are the vertices of the image?

STEP 1   Multiply each coordinate of the vertices of △ABC by 3 to find the vertices of the dilated image.

$$△ABC \ (x, y) \rightarrow (3x, 3y) \ △A'B'C'$$

$$A(1, 1) \rightarrow A'(1 \cdot 3, 1 \cdot 3) \rightarrow A'(3, 3)$$

$$B(3, 1) \rightarrow B'(3 \cdot 3, 1 \cdot 3) \rightarrow B'(9, 3)$$

$$C(1, 3) \rightarrow C'(1 \cdot 3, 3 \cdot 3) \rightarrow C'(3, 9)$$

The vertices of the dilated image are $A'(3, 3)$, $B'(9, 3)$, and $C'(3, 9)$.
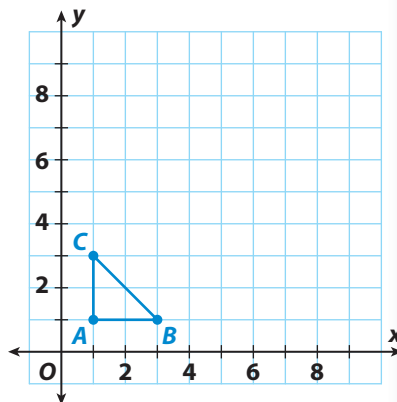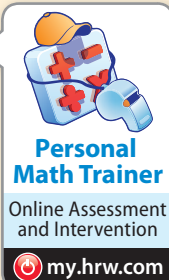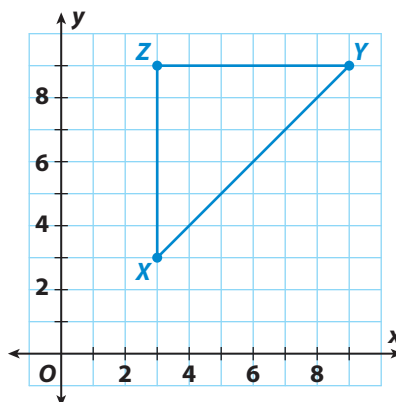
STEP 2   Graph the dilated image.
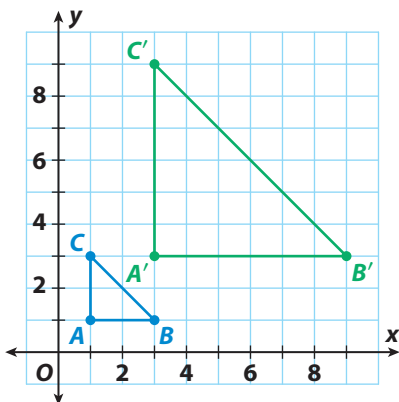
Math Talk

Mathematical Practices

Describe how you can check graphically that you have drawn the image triangle correctly.

YOUR TURN

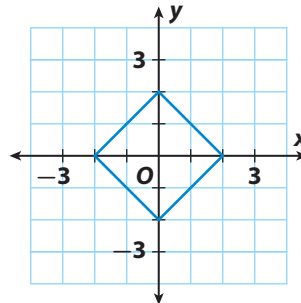**5.** Graph the image of △XYZ after a dilation with a scale factor of $\frac{1}{3}$ and the origin as its center. Then write an algebraic rule to describe the dilation.

1. The grid shows a diamond-shaped preimage. Write the coordinates of the vertices of the preimage in the first column of the table. Then apply the dilation $(x, y) \rightarrow \left(\frac{3}{2}x, \frac{3}{2}y\right)$ and write the coordinates of the vertices of the image in the second column. Sketch the image of the figure after the dilation. **(Explore Activities 1 and 2)**
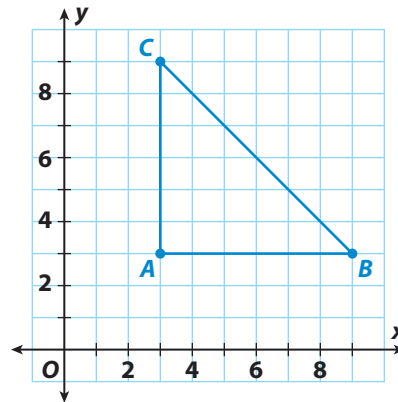
Preimage	Image
(2, 0)	(3, 0)

Graph the image of each figure after a dilation with the origin as its center and the given scale factor. Then write an algebraic rule to describe the dilation. (Example 1)

2. scale factor of 1.5
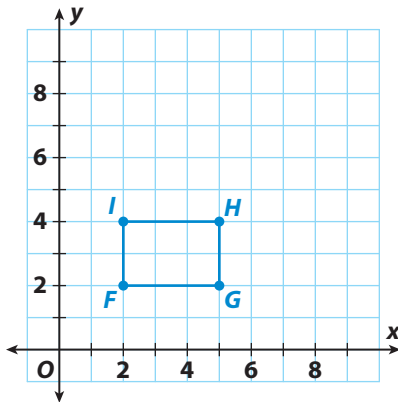
3. scale factor of $\frac{1}{3}$

_____

_____

? ESSENTIAL QUESTION CHECK-IN

4. A dilation of $(x, y) \rightarrow (kx, ky)$ when $0 < k < 1$ has what effect on the figure? What is the effect on the figure when $k > 1$?

_____

_____

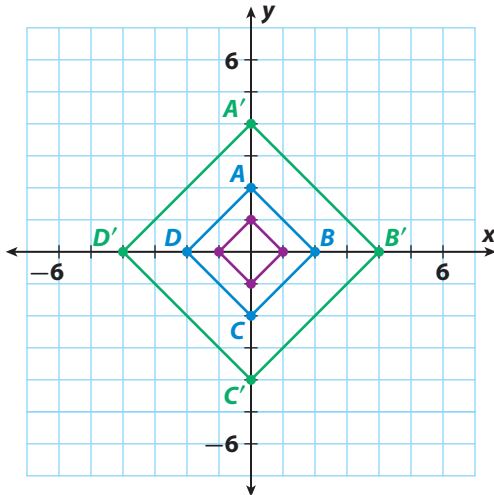_____

10.2 Independent Practice

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5. The blue square is the preimage. Write two algebraic representations, one for the dilation to the green square and one for the dilation to the purple square.

_____

_____

6. Critical Thinking A triangle has vertices $A(-5, -4)$, $B(2, 6)$, and $C(4, -3)$. The center of dilation is the origin and $(x, y) \rightarrow (3x, 3y)$. What are the vertices of the dilated image?

_____

_____

7. Critical Thinking $M'N'O'P'$ has vertices at $M'(3, 4)$, $N'(6, 4)$, $O'(6, 7)$, and $P'(3, 7)$. The center of dilation is the origin. $MNOP$ has vertices at $M(4.5, 6)$, $N(9, 6)$, $O'(9, 10.5)$, and $P'(4.5, 10.5)$. What is the algebraic representation of this dilation?

_____

_____

8. Critical Thinking A dilation with center $(0,0)$ and scale factor k is applied to a polygon. What dilation can you apply to the image to return it to the original preimage?

_____

9. Represent Real-World Problems The blueprints for a new house are scaled so that $\frac{1}{4}$ inch equals 1 foot. The blueprint is the preimage and the house is the dilated image. The blueprints are plotted on a coordinate plane.

**a.** What is the scale factor in terms of inches to inches?

_____

b. One inch on the blueprint represents how many inches in the actual house? How many feet?

_____

c. Write the algebraic representation of the dilation from the blueprint to the house.

_____

d. A rectangular room has coordinates $Q(2, 2)$, $R(7, 2)$, $S(7, 5)$, and $T(2, 5)$ on the blueprint. The homeowner wants this room to be 25% larger. What are the coordinates of the new room?

_____

_____

e. What are the dimensions of the new room, in inches, on the blueprint? What will the dimensions of the new room be, in feet, in the new house?

_____

_____

_____

_____

10. Write the algebraic representation of the dilation shown.

_____
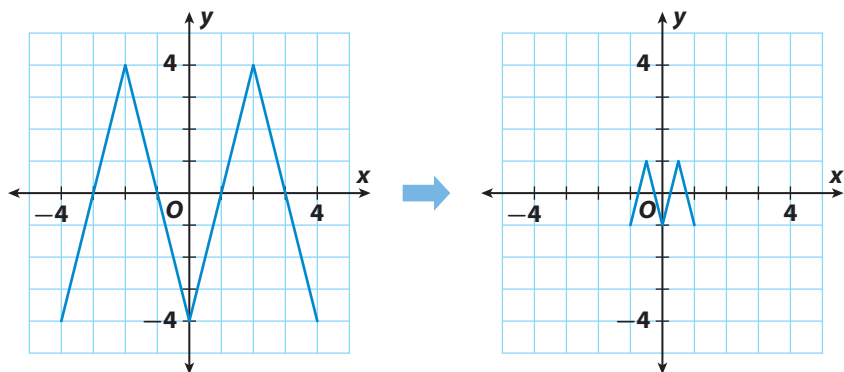
11. Critique Reasoning The set for a school play needs a replica of a historic building painted on a backdrop that is 20 feet long and 16 feet high. The actual building measures 400 feet long and 320 feet high. A stage crewmember writes $(x, y) \rightarrow \left(\frac{1}{12}x, \frac{1}{12}y\right)$ to represent the dilation. Is the crewmember's calculation correct if the painted replica is to cover the entire backdrop? Explain.

_____

_____

12. Communicate Mathematical Ideas Explain what each of these algebraic transformations does to a figure.

a. $(x, y) \rightarrow (y, -x)$ _____

b. $(x, y) \rightarrow (-x, -y)$ _____

c. $(x, y) \rightarrow (x, 2y)$ _____

_____

d. $(x, y) \rightarrow \left(\frac{2}{3}x, y\right)$ _____

_____

e. $(x, y) \rightarrow (0.5x, 1.5y)$ _____

_____

13. Communicate Mathematical Ideas Triangle _ABC_ has coordinates _A_(1, 5), _B_(−2, 1), and _C_(−2, 4). Sketch triangle _ABC_ and _A′B′C′_ for the dilation $(x, y) \rightarrow (-2x, -2y)$. What is the effect of a negative scale factor?

_____

_____

Work Area

LESSON
10.3 Similar Figures

FL 8.G.1.4

Understand that a . . . figure is similar to another if the second can be obtained . . . by a sequence of rotations, reflections, translations, and dilations; given two similar . . . figures, describe a sequence that exhibits the similarity between them.

? ESSENTIAL QUESTION

What is the connection between transformations and similar figures?
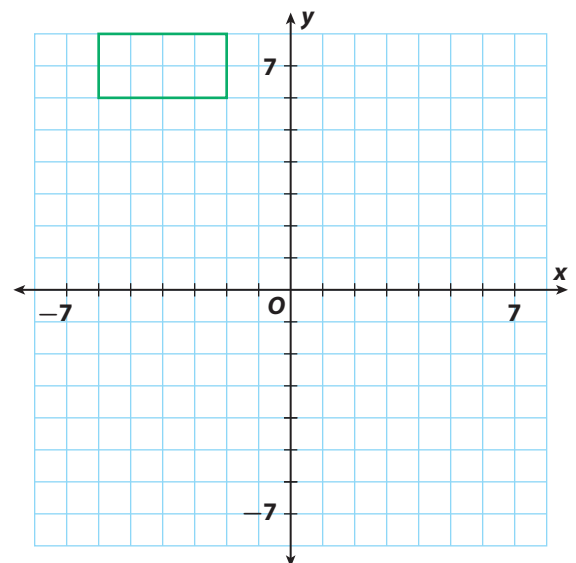
EXPLORE ACTIVITY FL 8.G.1.4

Combining Transformations with Dilations

When creating an animation, figures need to be translated, reflected, rotated, and sometimes dilated. As an example of this, apply the indicated sequence of transformations to the rectangle. Each transformation is applied to the image of the previous transformation, not to the original figure. Label each image with the letter of the transformation applied.

A $(x, y) \rightarrow (x + 7, y - 2)$

B $(x, y) \rightarrow (x, -y)$

C rotation 90° clockwise around the origin

D $(x, y) \rightarrow (x + 5, y + 3)$

E $(x, y) \rightarrow (3x, 3y)$

F List the coordinates of the vertices of rectangle E.

_____

G Compare the following attributes of rectangle E to those of the original figure.

Shape	
Size	
Angle Measures	

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Reflect

1. Which transformation represents the dilation? How can you tell?

   _____

   _____

2. A sequence of transformations containing a single dilation is applied to a figure. Are the original figure and its final image congruent? Explain.

   _____

   _____

Math On the Spot

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My Notes

Similar Figures

Two figures are **similar** if one can be obtained from the other by a sequence of translations, reflections, rotations, and dilations. Similar figures have the same shape but may be different sizes.

When you are told that two figures are similar, there must be a sequence of translations, reflections, rotations, and/or dilations that can transform one to the other.

EXAMPLE 1   FL   8.G.1.4

A Identify a sequence of transformations that will transform figure *A* into figure *B*. Tell whether the figures are congruent. Tell whether they are similar.
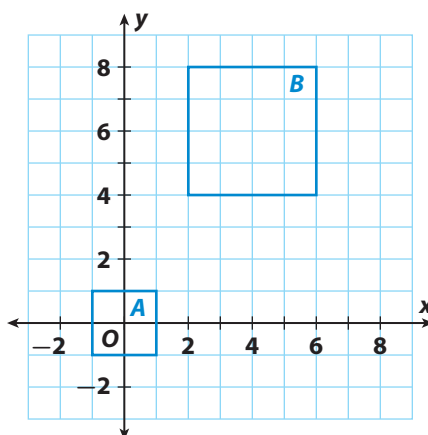
Both figures are squares whose orientations are the same, so no reflection or rotation is needed. Figure *B* has sides twice as long as figure *A*, so a dilation with a scale factor of 2 is needed. Figure *B* is moved to the right and above figure *A*, so a translation is needed. A sequence of transformations that will accomplish this is a dilation by a scale factor of 2 centered at the origin followed by the translation $(x, y) \rightarrow (x + 4, y + 6)$. The figures are not congruent, but they are similar.

B Identify a sequence of transformations that will transform figure *C* into figure *D*. Include a reflection. Tell whether the figures are congruent. Tell whether they are similar.
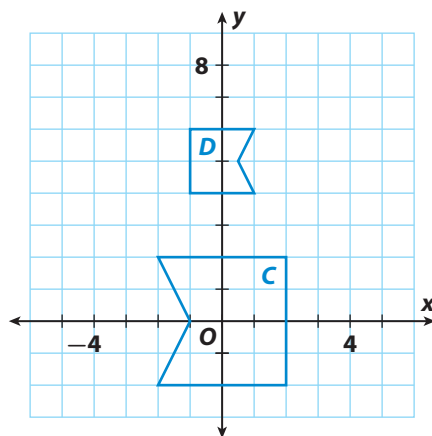
The orientation of figure *D* is reversed from that of figure *C*, so a reflection over the *y*-axis is needed. Figure *D* has sides that are half as long as figure *C*, so a dilation with a scale factor of $\frac{1}{2}$ is needed. Figure *D* is moved above figure *C*, so a translation is needed. A sequence of transformations that will accomplish this is a dilation by a scale factor of $\frac{1}{2}$ centered at the origin, followed by the reflection $(x, y) \rightarrow (-x, y)$, followed by the translation $(x, y) \rightarrow (x, y + 5)$. The figures are not congruent, but they are similar.

C Identify a sequence of transformations that will transform figure *C* into figure *D*. Include a rotation.

The orientation of figure *D* is reversed from that of figure *C*, so a rotation of 180° is needed. Figure *D* has sides that are half as long as figure *C*, so a dilation with a scale factor of $\frac{1}{2}$ is needed. Figure *D* is moved above figure *C*, so a translation is needed. A sequence of transformations that will accomplish this is a rotation of 180° about the origin, followed by a dilation by a scale factor of $\frac{1}{2}$ centered at the origin, followed by the translation $(x, y) \rightarrow (x, y + 5)$.

Math Talk
Mathematical Practices

A figure and its image have different sizes and orientations. What do you know about the sequence of transformations that generated the image?

YOUR TURN

3. Look again at the Explore Activity. Start with the original figure. Create a new sequence of transformations that will yield figure *E*, the final image. Your transformations do not need to produce the images in the same order in which they originally appeared.
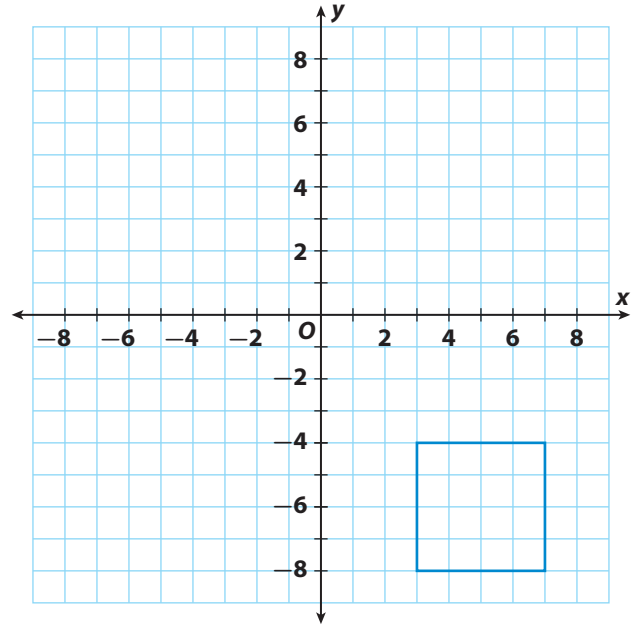
_____

_____

_____

Personal Math Trainer

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1. Apply the indicated sequence of transformations to the square. Apply each transformation to the image of the previous transformation. Label each image with the letter of the transformation applied. **(Explore Activity)**

A $(x, y) \rightarrow (-x, y)$

B Rotate the square 180° around the origin.

C $(x, y) \rightarrow (x - 5, y - 6)$

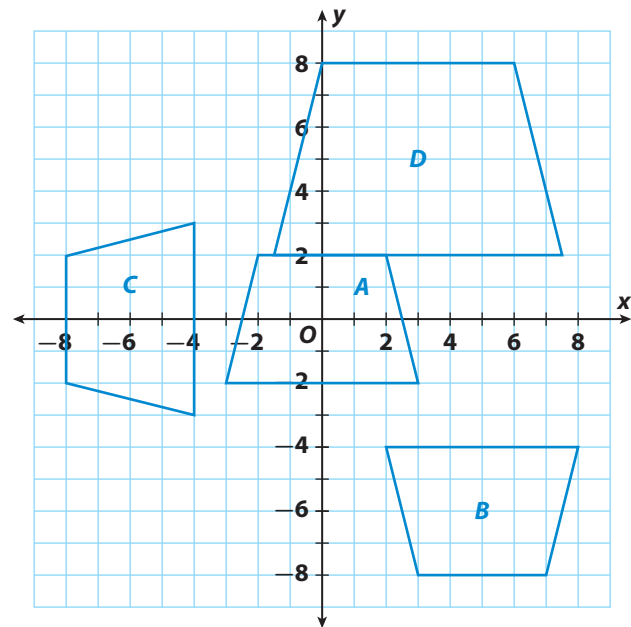**D** $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$

Identify a sequence of two transformations that will transform figure A into the given figure. **(Example 1)**

2. figure B

_____

_____

3. figure C

_____

_____

4. figure D

_____

_____

? ESSENTIAL QUESTION CHECK-IN

5. If two figures are similar but not congruent, what do you know about the sequence of transformations used to create one from the other?

_____

_____

10.3 Independent Practice

FL 8.G.1.4

6. A designer creates a drawing of a triangular sign on centimeter grid paper for a new business. The drawing has sides measuring 6 cm, 8 cm, and 10 cm, and angles measuring 37°, 53°, and 90°. To create the actual sign shown, the drawing must be dilated using a scale factor of 40.

Jan's Café

a. Find the lengths of the sides of the actual sign.

_____

b. Find the angle measures of the actual sign.

_____

c. The drawing has the hypotenuse on the bottom. The business owner would like it on the top. Describe two transformations that will do this.

_____

_____

d. The shorter leg of the drawing is currently on the left. The business owner wants it to remain on the left after the hypotenuse goes to the top. Which transformation in part c will accomplish this?

_____

In Exercises 7–10, the transformation of a figure into its image is described. Describe the transformations that will transform the image back into the original figure. Then write them algebraically.

7. The figure is reflected across the *x*-axis and dilated by a scale factor of 3.

_____

_____

8. The figure is dilated by a scale factor of 0.5 and translated 6 units left and 3 units up.

_____

_____

9. The figure is dilated by a scale factor of 5 and rotated 90° clockwise.

_____

_____

**10.** The figure is reflected across the *y*-axis and dilated by a scale factor of 4.

_____

_____

Work Area

11. Draw Conclusions A figure undergoes a sequence of transformations that include dilations. The figure and its final image are congruent. Explain how this can happen.

_____

_____

_____

12. Multistep As with geometric figures, graphs can be transformed through translations, reflections, rotations, and dilations. The equation $y = x$ is graphed at the right. Describe how the graph is changed through each of the following transformations.
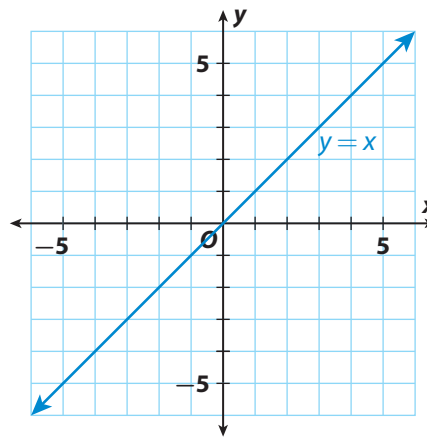
a. a dilation by a scale factor of 4, producing the graph of the equation $y = 4x$

_____

_____

_____

b. a translation producing the graph of the equation $y = x - 3$.

_____

_____

c. a reflection across the *y*-axis

_____

_____

13. Justify Reasoning The graph of the line $y = x$ is dilated by a scale factor of 3 and then translated up 5 units. Is this the same as translating the graph up 5 units and then dilating by a scale factor of 3? Explain. How are the new graphs related?

_____

_____

332 Unit 4

Ready to Go On?

10.1 Properties of Dilations

Determine whether one figure is a dilation of the other. Justify your answer.
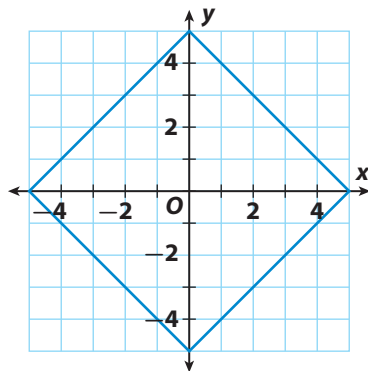
1. Triangle *XYZ* has angles measuring 54° and 29°. Triangle *X'Y'Z'* has angles measuring 29° and 92°.

_____

2. Quadrilateral *DEFG* has sides measuring 16 m, 28 m, 24 m, and 20 m. Quadrilateral *D'E'F'G'* has sides measuring 20 m, 35 m, 30 m, and 25 m.
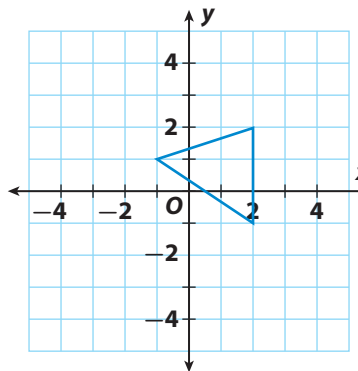
_____

_____

10.2 Algebraic Representations of Dilations

Dilate each figure with the origin as the center of dilation.

3. $(x, y) \rightarrow (0.8x, 0.8y)$

4. $(x, y) \rightarrow (2.5x, 2.5y)$

10.3 Similar Figures

5. Describe what happens to a figure when the given sequence of transformations is applied to it: $(x, y) \rightarrow (-x, y)$; $(x, y) \rightarrow (0.5x, 0.5y)$; $(x, y) \rightarrow (x - 2, y + 2)$
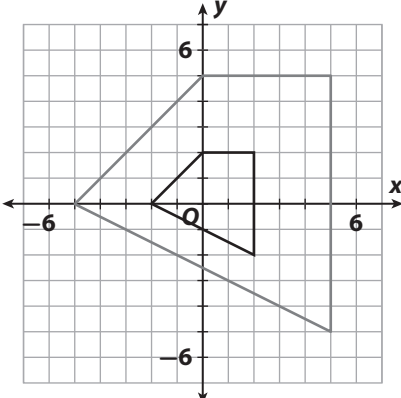
_____

_____

? ESSENTIAL QUESTION

6. How can you use dilations to solve real-world problems?

_____

Selected Response

1. A rectangle has vertices (6, 4), (2, 4), (6, –2), and (2, –2). What are the coordinates of the vertices of the image after a dilation with the origin as its center and a scale factor of 1.5?

Ⓐ (9, 6), (3, 6), (9, –3), (3, –3)

Ⓑ (3, 2), (1, 2), (3, –1), (1, –1)

Ⓒ (12, 8), (4, 8), (12, –4), (4, –4)

Ⓓ (15, 10), (5, 10), (15, –5), (5, –5)

2. Which represents the dilation shown where the black figure is the preimage?

Ⓐ $(x, y) \rightarrow (1.5x, 1.5y)$

Ⓑ $(x, y) \rightarrow (2.5x, 2.5y)$

Ⓒ $(x, y) \rightarrow (3x, 3y)$

Ⓓ $(x, y) \rightarrow (6x, 6y)$

3. Identify the sequence of transformations that will reflect a figure over the *x*-axis and then dilate it by a scale factor of 3.

Ⓐ $(x, y) \rightarrow (-x, y); (x, y) \rightarrow (3x, 3y)$

Ⓑ $(x, y) \rightarrow (-x, y); (x, y) \rightarrow (x, 3y)$

Ⓒ $(x, y) \rightarrow (x, -y); (x, y) \rightarrow (3x, y)$

Ⓓ $(x, y) \rightarrow (x, -y); (x, y) \rightarrow (3x, 3y)$
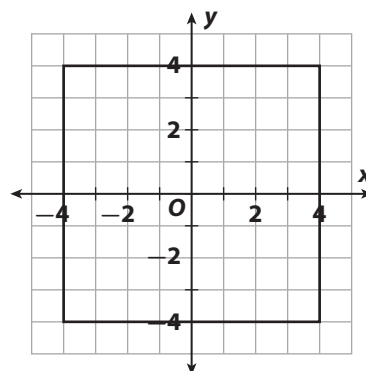
4. Solve $-a + 7 = 2a - 8$.

Ⓐ $a = -3$

Ⓑ $a = -\frac{1}{3}$

Ⓒ $a = 5$

Ⓓ $a = 15$

5. Which equation does **not** represent a line with an *x*-intercept of 3?

Ⓐ $y = -2x + 6$

Ⓑ $y = -\frac{1}{3}x + 1$

Ⓒ $y = \frac{2}{3}x - 2$

Ⓓ $y = 3x - 1$

Mini-Task

6. The square is dilated under the dilation $(x, y) \rightarrow (0.25x, 0.25y)$.

a. Graph the image. What are the coordinates?

_____

b. What is the length of a side of the image?

_____

c. What are the perimeter and area of the preimage?

_____

d. What are the perimeter and area of the image?

_____

Study Guide Review

MODULE 9 Transformations and Congruence

Key Vocabulary
center of rotation *(centro de rotación)*
congruent *(congruente)*
image *(imagen)*
line of reflection *(línea de reflexión)*
preimage *(imagen original)*
reflection *(reflexión)*
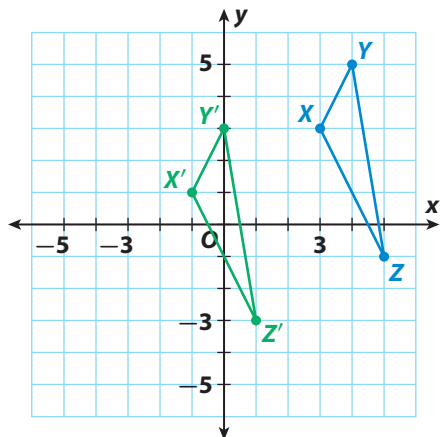rotation *(rotación)*
transformation *(transformación)*
translation *(traslación)*

? ESSENTIAL QUESTION

How can you use transformations and congruence to solve real-world problems?

EXAMPLE

Translate triangle *XYZ* left 4 units and down 2 units. Graph the image and label the vertices.

Translate the vertices by subtracting 4 from each *x*-coordinate and 2 from each *y*-coordinate. The new vertices are $X'(-1, 1)$, $Y'(0, 3)$, and $Z'(1, -3)$.

Connect the vertices to draw triangle $X'Y'Z'$.

EXERCISES

Perform the transformation shown. (Lessons 9.1, 9.2, 9.3)
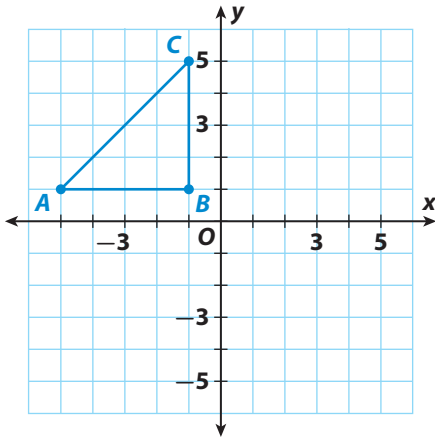
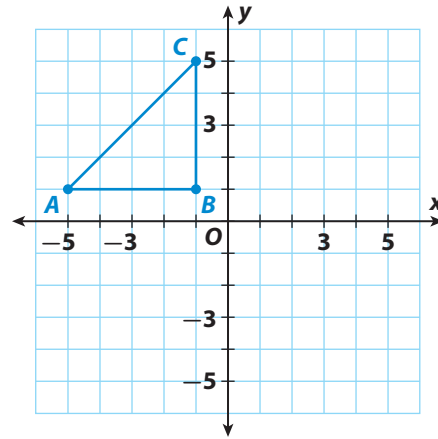**1.** Reflection over the *x*-axis
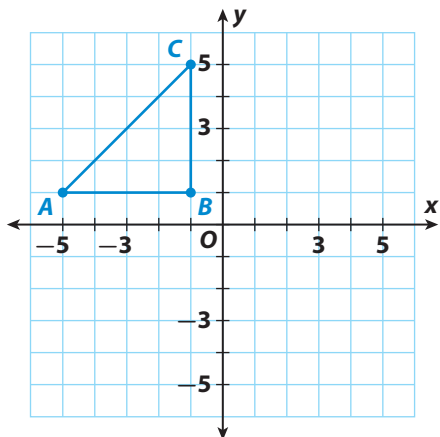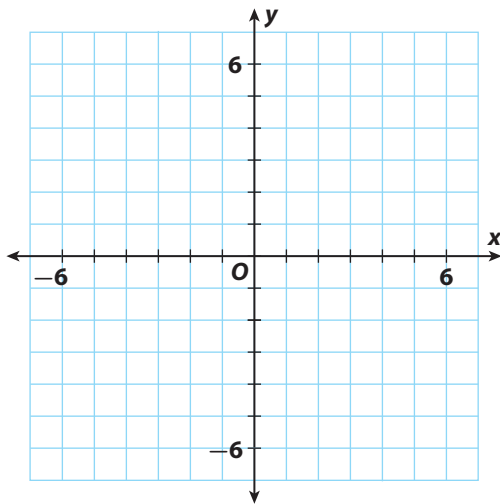

2. Translation 5 units right
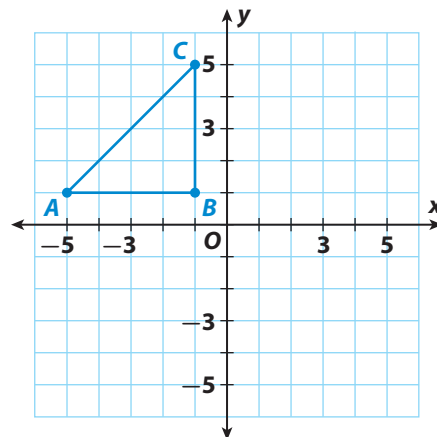
**3.** Rotation 90° counterclockwise about the origin

4. Translation 4 units right and 4 units down

5. Quadrilateral *ABCD* with vertices *A*(4, 4), *B*(5, 1), *C*(5, −1) and *D*(4, −2) is translated left 2 units and down 3 units. Graph the preimage and the image. (**Lesson 9.4**)

6. Triangle *ABC* with vertices *A*(1, 2), *B*(1, 4), and *C*(3, 3) is translated by $(x, y) \rightarrow (x - 4, y)$, and the result is reflected by $(x, y) \rightarrow (x, -y)$. Graph the preimage and the image. (**Lesson 9.5**)
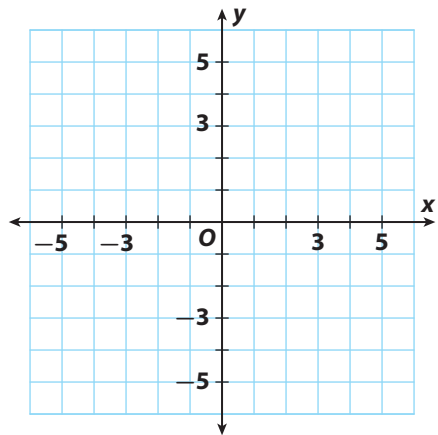
7. Triangle *RST* has vertices at (−8, 2), (−4, 0), and (−12, 8). Find the vertices after the triangle has been reflected over the *y*-axis. (**Lesson 9.4**)

_____

8. Triangle *XYZ* has vertices at (3, 7), (9, 14), and (12, −1). Find the vertices after the triangle has been rotated 180° about the origin. (**Lesson 9.4**)

_____

9. Triangle *MNP* has its vertices located at (−1, −4), (−2, −5), and (−3, −3). Find the vertices after the triangle has been reflected by $(x, y) \rightarrow (x, -y)$ and translated by $(x, y) \rightarrow (x + 6, y)$. (**Lesson 9.5**)
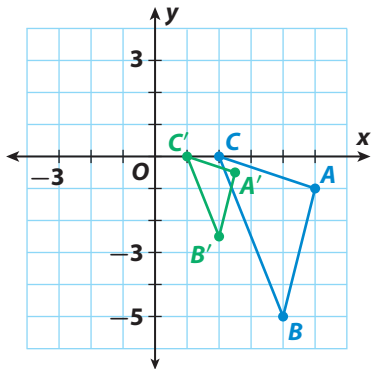
_____

Transformations and Similarity

? **ESSENTIAL QUESTION**

How can you use dilations, similarity, and proportionality to solve real-world problems?

EXAMPLE

Dilate triangle ABC with the origin as the center of dilation and scale factor $\frac{1}{2}$. Graph the dilated image.

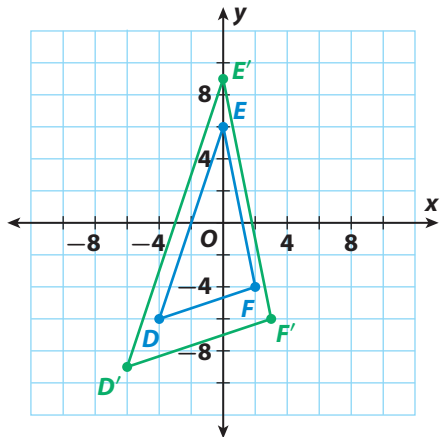Multiply each coordinate of the vertices of *ABC* by $\frac{1}{2}$ to find the vertices of the dilated image.

$A(5, -1) \rightarrow A'\left(5 \cdot \frac{1}{2}, -1 \cdot \frac{1}{2}\right) \rightarrow A'\left(2\frac{1}{2}, -\frac{1}{2}\right)$

$B(4, -5) \rightarrow B'\left(4 \cdot \frac{1}{2}, -5 \cdot \frac{1}{2}\right) \rightarrow B'\left(2, -2\frac{1}{2}\right)$

$C(2, 0) \rightarrow C'\left(2 \cdot \frac{1}{2}, 0 \cdot \frac{1}{2}\right) \rightarrow C'(1, 0)$

EXERCISES

1. For each pair of corresponding vertices, find the ratio of the *x*-coordinates and the ratio of the *y*-coordinates. **(Lesson 10.1)**

Ratio of *x*-coordinates: _____

Ratio of *y*-coordinates: _____

What is the scale factor of the dilation? _____

2. Rectangle *WXYZ* has vertices at $(-2, -1)$, $(-2, 1)$, $(2, -1)$, and $(2, 1)$. It is first dilated by $(x, y) \rightarrow (2x, 2y)$, and then translated by $(x, y) \rightarrow (x, y + 3)$. **(Lesson 10.3)**

a. What are the vertices of the image? _____

b. Are the preimage and image congruent? Are they similar? Explain.

_____

_____

Dilate each figure with the origin as the center of the dilation. List the vertices of the dilated figure then graph the figure. (Lesson 10.2)

3. $(x, y) \rightarrow \left(\frac{1}{4}x, \frac{1}{4}y\right)$

4. $(x, y) \rightarrow (2x, 2y)$

Unit 4 Performance Tasks

1. CAREERS IN MATH **Contractor** Fernando is expanding his dog's play yard. The original yard has a fence represented by rectangle *LMNO* on the coordinate plane. Fernando hires a contractor to construct a new fence that should enclose 6 times as much area as the current fence. The shape of the fence must remain the same. The contractor constructs the fence shown by rectangle *L′M′N′O′*.
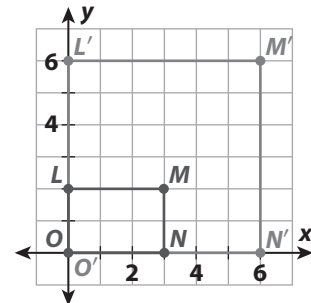
a. Did the contractor increase the area by the amount Fernando wanted? Explain.

_____

_____

b. Does the new fence maintain the shape of the old fence? How do you know?

_____

_____

2. A sail for a sailboat is represented by a triangle on the coordinate plane with vertices (0, 0), (5, 0), and (5, 4). The triangle is dilated by a scale factor of 1.5 with the origin as the center of dilation. Find the coordinates of the dilated triangle. Are the triangles similar? Explain.

_____

_____

Selected Response

1. What would be the orientation of the figure below after a reflection over the *x*-axis?

(A)

(B)

(C)

(D)
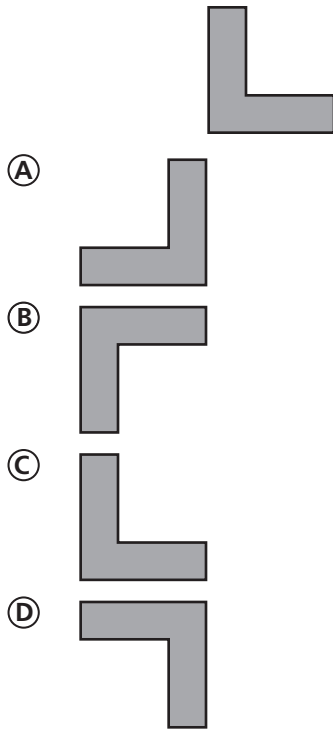
2. A triangle with coordinates (4, 2), (0, −3), and (−5, 3) is translated 5 units right and rotated 180° about the origin. What are the coordinates of its image?

(A) (9, 2), (−1, −2), (5, −7)

(B) (−10, 3), (−1, 2), (−5, −3)

(C) (2, −1), (−3, −5), (3, −10)

(D) (−9, −2), (−5, 3), (0, −3)

3. Quadrilateral *LMNP* has sides measuring 16, 28, 12, and 32. Which could be the side lengths of a dilation of *LMNP*?

(A) 24, 40, 18, 90

(B) 32, 60, 24, 65

(C) 20, 35, 15, 40

(D) 40, 70, 30, 75

4. The table below represents which equation?

x	−1	0	1	2
y	1	−2	−5	−8

(A) $y = x + 2$

(B) $y = -x$

(C) $y = 3x + 6$

(D) $y = -3x - 2$

5. Which of the following is **not** true of a trapezoid that has been translated 8 units down?

(A) The new trapezoid is the same size as the original trapezoid.

(B) The new trapezoid is the same shape as the original trapezoid.

(C) The new trapezoid is in the same orientation as the original trapezoid.

(D) The *y*-coordinates of the new trapezoid are the same as the *y*-coordinates of the original trapezoid.

6. Which represents a reduction?

(A) $(x, y) \rightarrow (0.9x, 0.9y)$

(B) $(x, y) \rightarrow (1.4x, 1.4y)$

(C) $(x, y) \rightarrow (0.7x, 0.3y)$

(D) $(x, y) \rightarrow (2.5x, 2.5y)$

7. Which is the solution for $4(x + 1) = 2(3x - 2)$?

(A) $x = -4$

(B) $x = -1$

(C) $x = 0$

(D) $x = 4$

8. A rectangle has vertices (8, 6), (4, 6), (8, −4), and (4, −4). What are the coordinates after dilating from the origin by a scale factor of 1.5?

Ⓐ (9, 6), (3, 6), (9, −3), (3, −3)

Ⓑ (10, 8), (5, 8), (10, −5), (5, −5)

Ⓒ (16, 12), (8, 12), (16, −8), (8, −8)

Ⓓ (12, 9), (6, 9), (12, −6), (6, −6)

> **Hot Tip!** Make sure you look at all answer choices before making your decision. Try substituting each answer choice into the problem if you are unsure of the answer.

9. Two apples plus four bananas cost $2.00. An apple costs twice as much as a banana. Using the equations $2a + 4b = 2.00$ and $a = 2b$, where a is the cost of one apple and b is the cost of one banana, what are a and b?

Ⓐ $a = \$0.25; b = \$0.25$

Ⓑ $a = \$0.25; b = \$0.50$

Ⓒ $a = \$0.50; b = \$0.25$

Ⓓ $a = \$0.50; b = \$0.50$

10. Which statement is false?

Ⓐ No integers are irrational numbers.

Ⓑ All whole numbers are integers.

Ⓒ No real numbers are rational numbers.

Ⓓ All integers greater than or equal to 0 are whole numbers.
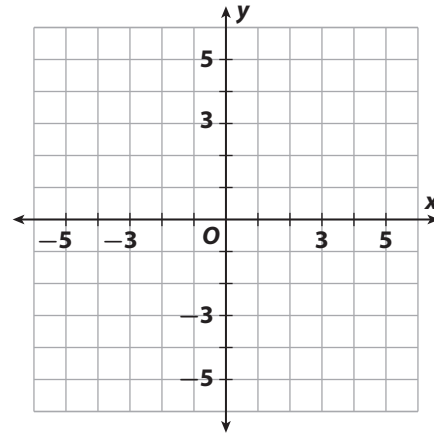
11. Consider the system of equations $3x + 4y = 2$ and $2x − 4y = 8$. Which is its solution?

Ⓐ $x = −1, y = −2$

Ⓑ $x = 1, y = 2$

Ⓒ $x = −2, y = 1$

Ⓓ $x = 2, y = −1$

12. A triangle with vertices (−2, −3), (−4, 0), and (0, 0) is congruent to a second triangle located in quadrant I with two of its vertices at (3, 2) and (1, 5).

a. Graph the two triangles on the same coordinate grid.

b. What are the coordinates of the third vertex of the second triangle?

_____

13. Tamiko is planning a stone wall shaped like a triangle, with vertices at (−1, −2), (2, 2), and (−2, 2) on a coordinate grid. She plans to add a second wall, in the same shape, enclosing the first wall, with the origin as the center of dilation. The vertices of the second wall are (−3, −6), (6, 6), and (−6, 6).

a. What scale factor did Tamiko use for the second wall?

_____

b. Are the two walls similar? Explain.

_____

_____

_____

_____